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Nonuniform Hyperbolicity: Dynamics of Systems
with Nonzero Lyapunov Exponents

To Clàudia

and

To Natasha, Irishka and Lenchka

for their patience, encouragement and inspiration

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Preface

Writing this book was a long-term project that has taken several years, and at the early stages Anatole Katok's participation was crucial. He provided us with the text of his unpublished notes with Leonardo Mendoza that served as the basis for the first draft of Chapters 1–5 and parts of Chapters 6–7 of the book. He also fully participated in designing the content of the book in its present form.

In this book we present a self-contained and sufficiently complete description of the modern nonuniform hyperbolicity theory, that is, the theory of dynamical systems whose Lyapunov exponents are not zero. The reader will find all the core results of the theory as well as a good account of its recent developments.

The nonuniform hyperbolicity theory is rich in wonderful ideas and sophisticated techniques, which are widely used in many areas of dynamical systems as well as other areas of mathematics and beyond. The nonuniform hyperbolicity theory is very popular and finds a lot of applications outside mathematics – in physics, biology, engineering, and so on.

Despite (or should we say because of) a tremendous amount of research on the subject, there have been relatively few attempts to summarize and unify the results of the theory in a single manuscript or a survey (see the books [110, 139, 179], the surveys [18, 137, 175], and the lectures [19, 28]). This book is meant to cover this gap. It can be used as a reference book for the theory or as a supporting material for an advanced course on dynamical systems. During the long course of working on this book, we first produced its baby version [20] where we described the core results of the theory and some principal examples, and then we wrote the survey [21] where we presented the contemporary status of the theory.

Since the beginning of the 1970s, the nonuniform hyperbolicity theory has emerged as an independent discipline lying in the heart of the modern theory of dynamical systems. It studies both conservative (volume preserving) and dissipative systems, deterministic as well as random dynamical systems, discrete and continuous-time systems, in addition to cocycles and group actions. The results of this theory have found their way in geometry (e.g., in the study of geodesic flows

and Teichmüller flows), in rigidity theory, in the study of some partial differential equations (e.g., the Schrödinger equation and some reaction-diffusion equations), and in the theory of “chaotic” billiards.

Writing a book of such a scope can be deemed as a daunting task and we therefore had to select topics so that personal taste, clearly biased toward our own interests, entered in our choices. As a result, some interesting topics are barely mentioned or not covered at all. In particular, we do not consider random dynamical systems referring the reader to the books [9, 112, 129] and the survey [113] nor dynamical systems with singularities (see the book [110]), in particular, leaving aside the rich theory of chaotic billiards. We restrict ourselves to the case of invertible dynamical systems and thus the theory of nonuniformly expanding maps is not discussed here (see the survey [132]) nor do we include one-dimensional chaotic maps (e.g., the logistic family, see [94]). We touch upon some recent results on Hénon-like attractors related to the study of Sinai–Ruelle–Bowen measures but we do not go deep into the theory of these attractors (see the survey [133]). We mention some results on hyperbolic group actions and refer the reader to [71] for a more complete account.

All the principal results of the nonuniform hyperbolicity theory are presented in the book with complete proofs, although some other results are included without proofs for the sake of completeness.

Most chapters of the book end with notes where the reader can find some remarks of historical and bibliographical nature, comments on some related results, and references for further reading. In no way these notes are meant to present a significant account of the history of the subject or a sufficiently complete list of references.

Acknowledgments

While working on the book, several people helped us in various ways and it is our great pleasure to acknowledge their contributions.

As we mentioned above, it is impossible to overestimate the contribution of Anatole Katok whom we heartily thank for his constant support and guidance.

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It is our pleasure to thank Anton Zorich for useful comments on Section 12.5.

We are grateful to Claudia Valls who carefully read several chapters of the book and made many remarks that helped us clarify the presentation.

When the first draft of the book was ready, we sent it to several experts in the field asking for their opinions and comments. We are grateful to Dima Dolgopyat, François Ledrappier, Mark Pollicott, and Federico Rodríguez Hertz for innumerable constructive suggestions that helped us improve the content and presentation of some results of the book as well as extend and enhance the bibliography.

Our special thanks go to Boris Hasselblatt who thoroughly examined the draft

and pointed out many places in the book that needed additional work. Reflecting on his comments we introduced many changes improving the style and exposition of the material; we also added more informal discussions, hopefully making the book more reader friendly.

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Introduction

The goal of this book is to present smooth ergodic theory from a contemporary point of view. Among other things this theory provides a rigorous mathematical foundation for the phenomenon known as *deterministic chaos* – a term coined by Yorke – the appearance of highly irregular, unpredictable, “chaotic” motions in pure deterministic dynamical systems. The main idea beyond this phenomenon is that one can deduce a sufficiently complete description of topological and ergodic properties of the system from relatively weak requirements on its local behavior, known as *nonuniform hyperbolicity conditions*: the reason this theory is also called nonuniform hyperbolicity theory.

It originated in the seminal works of Lyapunov [134] and Perron [164] on stability of solutions of ordinary differential equations. To determine whether a given solution is stable one proceeds as follows. First, the equation is linearized along the solution and then the stability of the zero solution of the corresponding nonautonomous linear differential equation is examined. There are several methods (due to Hadamard [79], Perron [165], Fenichel [70], and Irwin [92]) aimed at exhibiting stability of solutions via certain information on the linear system. The approach by Lyapunov uses a special real-valued function on the space of solutions of the linear system known as the *Lyapunov exponent*. It measures in the logarithmic scale the rate of convergence of solutions so that the zero solution is asymptotically exponentially stable along any subspace where the Lyapunov exponent is negative.

The Lyapunov exponent is arguable the best way to characterize stability: the requirement that the Lyapunov exponent is negative is the weakest one that still guarantees that solutions of the linear system eventually decay exponentially to zero. The price to pay is that stability of the zero solution in this weak sense does not necessarily imply stability of the original solution of the nonlinear equation. The latter can be ensured under an additional and quite subtle requirement known as the *Lyapunov–Perron regularity*.

Verifying this requirement for a given solution may be a very difficult if not virtually impossible task, making verification more a principle than practical matter. This could deem the whole approach useless if not for an important particular case

when the differential equation is given by a vector field on a smooth compact Riemannian manifold. In this case, the celebrated Multiplicative Ergodic Theorem, also known as Oseledec's theorem, claims that a “typical” solution of the equation is Lyapunov–Perron regular, thus making the difficult task of checking the regularity requirement unnecessary. Here “typical” means that the statement holds for almost every trajectory with respect to a finite Borel measure invariant under the flow generated by the vector field.

A principal application of Oseledec's theorem in the context of smooth dynamical systems is that the Lyapunov exponent alone can be used to characterize stability of trajectories. Building upon this idea, in the beginning of 1970s Pesin introduced the class of systems whose Lyapunov exponent is nonzero along almost every trajectory with respect to some *smooth invariant measure* (i.e., a measure, which is equivalent to the Riemannian volume) and then he developed the stability theory (constructing local and global stable and unstable manifolds; see Section 7.5), as well as described their ergodic properties (including ergodicity, K - and Bernoulli properties; see Chapter 9). The collection of these results is known as Pesin's theory (see [18]). A crucial manifestation of this theory is the *formula for the entropy* connecting the measure-theoretic entropy of the system with its Lyapunov exponent (see Chapter 10). It should be pointed out that these results require that the system is of class of smoothness $C^{1+\alpha}$ for some $\alpha > 0$ and that they may indeed fail if the system is only of class C^1 (see Section 7.8).

Unlike classical uniformly hyperbolic systems (i.e., Anosov or more general axiom A systems) where contractions and expansions are *uniform everywhere* on a compact invariant set, Pesin's theory deals with systems satisfying the substantially weaker requirement that contractions and expansions occur *asymptotically almost everywhere* with respect to a smooth invariant measure. Because this requirement is weak, there are no topological obstructions for the existence of such systems on any phase space. Indeed, any smooth compact Riemannian manifold (of dimension ≥ 2 in the discrete-time case and of dimension ≥ 3 in the continuous-time case) admits a volume preserving system whose Lyapunov exponent is nonzero almost everywhere (see Sections 11.4 and 11.5). It is therefore remarkable that such a weak requirement ensures highly nontrivial ergodic and topological properties of the system.

A small perturbation of a diffeomorphism with nonzero Lyapunov exponents (in the C^r topology, $r > 1$) may not bear the same properties – the price to pay for the great generality of the nonuniformly hyperbolic theory. However, experts believe that nonuniformly hyperbolic *conservative* systems (i.e., systems preserving a smooth measure, in particular, volume preserving) are *typical* in some sense. This is reflected in the following conjectures: (We consider the case of systems with discrete time.)

1. Let f be a C^r , $r > 1$, volume preserving diffeomorphism of a smooth compact Riemannian manifold M . Assume that the Lyapunov exponent of f is nonzero

along almost every trajectory of f . Then there exists a neighborhood \mathcal{U} of f in the space of C^r volume preserving diffeomorphisms of M and a residual subset $\mathcal{A} \subset \mathcal{U}$ such that for every $g \in \mathcal{A}$ the Lyapunov exponent of g is nonzero along every orbit in a subset of positive volume.

2. Let f be a C^r , $r > 1$, volume preserving diffeomorphism of a smooth compact Riemannian manifold M . Then arbitrarily close to f in the space of C^r volume preserving diffeomorphisms of M , there exists a diffeomorphism g whose Lyapunov exponent is nonzero along every orbit in a subset of positive volume.

We stress that the assumption $r > 1$ is crucial as the conjectures fail if $r = 1$ due to a recent result of Bochi and Viana [28]. So far there has been little progress in solving these conjectures (see Section 11.7). On the positive side, crucial results on genericity of hyperbolic cocycles over dynamical systems have been recently obtained by Viana [221].

A persistent obstruction to nonuniform hyperbolicity is presence of elliptic behavior (see [232, 233]). For example, for area preserving surface diffeomorphisms, as predicted by KAM theory, elliptic islands survive under small perturbations of the system. Numerical studies of such maps suggest that in this case elliptic islands coexist with what appears to be a “chaotic sea” – an ergodic component of positive area with nonzero Lyapunov exponents (see [135, 136]). In fact, one often considers a one-parameter family of area preserving surface diffeomorphisms, which starts from a completely integrable (nonchaotic) system and evolves eventually into a completely hyperbolic (chaotic) one demonstrating, for intermediate values of the parameter, the appearance of elliptic islands gradually giving way to a “chaotic sea”. For billiard dynamical systems, coexistence of elliptic and hyperbolic behavior has been shown for the so-called “mushroom billiards” (see [41]). In the category of smooth maps, establishing coexistence is arguably one of the most difficult problems in the theory of dynamical systems. A simple but somewhat “artificial” example of coexistence was constructed in [183] (see also [130] and Section 6.6; for a more elaborate construction see [90]). Much more complicated examples where coexistence is expected are (1) the famous standard map (also known as the Chirikov–Taylor map; see [51] and [188, Section 8.5]) and (2) automorphisms of real $K3$ surfaces (see [151]).

The requirement that the Lyapunov exponent is nonzero along almost every trajectory with respect to an invariant Borel probability measure – such a measure is said to be *hyperbolic* – is equivalent to the fact that the system is nonuniformly hyperbolic. Thus nonuniform hyperbolicity can be viewed as presence of hyperbolic invariant measures leading to challenging problems of studying ergodic and topological properties of general (not necessarily smooth) hyperbolic measures as well as of constructing some *natural* hyperbolic measures.

A general hyperbolic measure does not have “good” ergodic properties. (Simply note that *any* invariant measure on a horseshoe is hyperbolic.) It is therefore quite remarkable that hyperbolic measures have abundance of topological proper-

ties whose study was initiated in the work of Katok [101] (see Chapters 14 and 15). For example, the set of hyperbolic periodic orbits is dense in the support of the measure. Surprisingly, general hyperbolic measures asymptotically have local product structure (similar to the one of Gibbs measures on horseshoes) and one can compute their Hausdorff dimension and entropy. The formula for the entropy of a general hyperbolic measure due to Ledrappier and L.-S. Young is a substantial generalization of the entropy formula for smooth hyperbolic measures but unlike the latter, it involves quite subtle characteristics of the measure other than the Lyapunov exponent.

Smooth measures form an important yet particular case of natural hyperbolic measures. The latter were introduced by Ledrappier as an extension to nonuniformly hyperbolic systems of the Sinai–Ruelle–Bowen (SRB) measures for classical uniformly hyperbolic attractors (see Chapter 13). These *generalized* SRB measures describe the limit distribution of the time averages of continuous functions along forward orbits for a set of initial points of positive Lebesgue measure in a small neighborhood of the attractor. According to a result by Ledrappier and Strelcyn, these measures can be characterized as being the only measures for which the entropy formula of Pesin holds. Ledrappier showed that the methods used in studying ergodic properties of smooth hyperbolic measures can be adjusted to describe ergodic properties of SRB measures.

Constructing SRB measures for nonuniformly hyperbolic systems is a difficult problem. Beyond uniform hyperbolicity, there are very few examples, of which best known are Hénon-like attractors, where existence of SRB measures was rigorously shown. L.-S. Young has introduced a class of dynamical systems with nonzero Lyapunov exponents, which admit the so-called Young’s tower. For these systems, she established existence of SRB measures (see Section 13.3).

The recent theory of Hénon-like diffeomorphisms (see [25, 26, 219, 222, 223]) suggests the following approach to the genericity problem for nonuniformly hyperbolic *dissipative* systems: given a one-parameter family of C^2 diffeomorphisms f_a , $a \in [\alpha, \beta]$ with a trapping region R (i.e., R is an open set for which $\overline{f_a(R)} \subset R$ for any $a \in [\alpha, \beta]$), there exists a set $A \subset [\alpha, \beta]$ of positive Lebesgue measure such that for every $a \in A$, the diffeomorphism f_a possesses an SRB measure supported on the attractor $\Lambda_a = \bigcap_{n>0} f_a^n(R)$.

Evaluating Lyapunov exponents by a computer is a relatively easy procedure and in many models in science, the absence of zero exponents can be shown numerically. This is often viewed as a convincing evidence that the system under investigation exhibits chaotic behavior. In mathematics, several “artificial” examples of systems with nonzero exponents have been constructed (and the reader can find most of them in Chapter 6) and for some interesting “natural” dynamical systems (e.g., geodesic flows on nonpositively curved manifolds and Teichmüller geodesic flows; see Chapter 12) absence of zero exponents have been shown. In addition, various powerful methods have been developed (e.g., cone and Lyapunov func-

tion techniques; see Chapter 4) that allow one to verify whether a given dynamical system has some positive Lyapunov exponents.

Many results of the nonuniform hyperbolicity theory hold in greater generality than for actions of single dynamical systems and wherever possible we describe the theory with this view in mind. For example, the linear hyperbolicity theory (including the theory of Lyapunov exponents and its principal result – the Multiplicative Ergodic Theorem) is presented for linear cocycles over dynamical systems (or even over higher-rank Abelian actions), and the stable manifold theory is developed for sequences of diffeomorphisms. Even in the case of an action of a single dynamical system, we consider a more general case of nonuniform *partial* hyperbolicity where the requirement that the values of the Lyapunov exponent are *all* nonzero is replaced by a weaker one that *some* of the values of the Lyapunov exponent are nonzero. Such generalizations require some more complicated techniques and tools from various areas of mathematics to be used and thus make the exposition more complicated but they substantially broaden applications and show the great power of the nonuniform hyperbolicity theory.

Bibliography

- [1] V. Alekseev, *Quasirandom dynamical systems. I. Quasirandom diffeomorphisms*, Math. USSR-Sb. **5** (1968), 73–128.
- [2] V. Alekseev, *Quasirandom dynamical systems. II. One-dimensional nonlinear vibrations in a periodically perturbed field*, Math. USSR-Sb. **6** (1968), 505–560.
- [3] V. Alekseev, *Quasirandom dynamical systems. III. Quasirandom vibrations of one-dimensional oscillators*, Math. USSR-Sb. **7** (1969), 1–43.
- [4] J. Alves, *SRB measures for nonhyperbolic systems with multidimensional expansion*, Ann. Sci. École Norm. Sup. (4) **33** (2000), 1–32.
- [5] J. Alves, C. Bonnatti and M. Viana, *SRB measures for partially hyperbolic systems whose central direction is mostly expanding*, Invent. Math. **140** (2000), 351–398.
- [6] D. Anosov, *Geodesic Flows on Closed Riemann Manifolds with Negative Curvature*, Proc. Steklov Inst. Math. **90** (1969), 1–235.
- [7] D. Anosov and Ya. Sinai, *Certain smooth ergodic systems*, Russian Math. Surveys **22** (1967), 103–167.
- [8] V. Araújo, *Non-zero Lyapunov exponents, no sign changes and Axiom A*, preprint.
- [9] L. Arnold, *Random Dynamical Systems*, Monographs in Mathematics, Springer, 1998.
- [10] V. Arnold and A. Avez, *Problèmes Ergodiques de la Mécanique Classique*, Monographies Internationales de Mathématiques Modernes 9, Gauthier-Villars, Paris, 1967.
- [11] A. Avila and J. Bochi, *A formula with some applications to the theory of Lyapunov exponents*, Israel J. Math. **131** (2002), 125–137.
- [12] A. Avila and M. Viana, *Simplicity of Lyapunov spectra: proof of the Zorich–Kontsevich conjecture*, Acta Math., to appear.
- [13] M. Babillot, *On the mixing property for hyperbolic systems*, Israel J. Math. **129** (2002), 61–76.
- [14] W. Ballmann, *Lectures on Spaces of Nonpositive Curvature*, with an appendix by M. Brin, Birkhäuser, 1995.
- [15] W. Ballmann and M. Brin, *On the ergodicity of geodesic flows*, Ergodic Theory Dynam. Systems **2** (1982), 311–315.
- [16] W. Ballmann, M. Brin and P. Eberlein, *Structure of manifolds of nonpositive curvature. I*, Ann. of Math. (2) **122** (1985), 171–203.
- [17] A. Baraviera and C. Bonatti, *Removing zero Lyapunov exponents*, Ergodic Theory Dynam. Systems **23** (2003), 1655–1670.
- [18] L. Barreira, *Pesin theory*, Encyclopaedia of Mathematics, Supplement Volume I, edited by M. Hazewinkel, Kluwer, 1997, pp. 406–411.
- [19] L. Barreira and Ya. Pesin, *Lectures on Lyapunov exponents and smooth ergodic theory*, in Smooth Ergodic Theory and Its Applications, edited by A. Katok, R. de la Llave, Ya. Pesin and H. Weiss, Proc. Symp. Pure Math. 69, Amer. Math. Soc., 2001, pp. 3–89.
- [20] L. Barreira and Ya. Pesin, *Lyapunov Exponents and Smooth Ergodic Theory*, University Lecture Series 23, Amer. Math. Soc., 2002.
- [21] L. Barreira and Ya. Pesin, *Smooth ergodic theory and nonuniformly hyperbolic dynamics*, with appendix by O. Sarig, in Handbook of Dynamical Systems 1B, edited by B. Hasselblatt and A. Katok, Elsevier, 2006, pp. 57–263.
- [22] L. Barreira, Ya. Pesin and J. Schmeling, *Dimension and product structure of hyperbolic measures*, Ann. of Math. (2) **149** (1999), 755–783.
- [23] L. Barreira and J. Schmeling, *Sets of “non-typical” points have full topological entropy and full Hausdorff dimension*, Israel J. Math. **116** (2000), 29–70.
- [24] L. Barreira and C. Silva, *Lyapunov exponents for continuous transformations and dimension theory*, Discrete Contin. Dyn. Syst. **13** (2005), 469–490.

- [25] M. Benedicks and L. Carleson, *The dynamics of the Hénon map*, Ann. of Math. (2) **133** (1991), 73–169.
- [26] M. Benedicks and L.-S. Young, *Sinai–Bowen–Ruelle measures for certain Hénon maps*, Invent. Math. **112** (1993), 541–576.
- [27] J. Bochi and M. Viana, *Pisa lectures on Lyapunov exponents*, in Dynamical Systems II, Pubbl. Cent. Ric. Mat. Ennio Giorgi, Scuola Norm. Sup., Pisa, 2003, pp. 23–47.
- [28] J. Bochi and M. Viana, *The Lyapunov exponents of generic volume-preserving and symplectic maps*, Ann. of Math. (2) **161** (2005), 1423–1485.
- [29] C. Bonatti, L. Díaz and R. Ures, *Minimality of strong stable and unstable foliations for partially hyperbolic diffeomorphisms*, J. Inst. Math. Jussieu **1** (2002), 513–541.
- [30] C. Bonatti, L. Díaz and M. Viana, *Dynamics beyond uniform hyperbolicity. A global geometric and probabilistic perspective*, Encyclopaedia of Mathematical Sciences 102, Mathematical Physics III, Springer, 2005.
- [31] C. Bonatti, C. Matheus, M. Viana and A. Wilkinson, *Abundance of stable ergodicity*, Comment. Math. Helv. **79** (2004), 753–757.
- [32] C. Bonatti and M. Viana, *SRB measures for partially hyperbolic systems whose central direction is mostly contracting*, Israel J. Math. **115** (2000), 157–193.
- [33] P. Bougerol, *Kalman filtering with random coefficients and contractions*, SIAM J. Control Optim. **31** (1993), 942–959.
- [34] R. Bowen, *Mixing Anosov flows*, Topology **15** (1976), 77–79.
- [35] M. Brin, *Topological transitivity of a certain class of dynamical systems, and flows of frames on manifolds of negative curvature*, Funct. Anal. Appl. **9** (1975), 8–16.
- [36] M. Brin, *The topology of group extensions of C-systems*, Mat. Zametki **18** (1975), 453–465.
- [37] M. Brin, *Bernoulli diffeomorphisms with nonzero exponents*, Ergodic Theory Dynam. Systems **1** (1981), 1–7.
- [38] M. Brin, *Hölder continuity of invariant distributions*, in Smooth Ergodic Theory and Its Applications, edited by A. Katok, R. de la Llave, Ya. Pesin and H. Weiss, Proc. Symp. Pure Math. 69, Amer. Math. Soc., 2001, pp. 99–101.
- [39] M. Brin, J. Feldman and A. Katok, *Bernoulli diffeomorphisms and group extensions of dynamical systems with non-zero characteristic exponents*, Ann. of Math. (2) **113** (1981), 159–179.
- [40] M. Brin and G. Stuck, *Introduction to Dynamical Systems*, Cambridge Univ. Press, 2002.
- [41] L. Bunimovich, *Mushrooms and other billiards with divided phase space*, Chaos **11** (2001), 802–808.
- [42] K. Burns, D. Dolgopyat and Ya. Pesin, *Partial hyperbolicity, Lyapunov exponents and stable ergodicity*, J. Stat. Phys. **108** (2002), 927–942.
- [43] K. Burns, D. Dolgopyat, Ya. Pesin and M. Pollicott *Stable ergodicity of partially hyperbolic attractors with nonzero exponents*, Pure and Applied Mathematics Quarterly, to appear.
- [44] K. Burns and M. Gerber, *Continuous invariant cone families and ergodicity of flows in dimension three*, Ergodic Theory Dynam. Systems **9** (1989), 19–25.
- [45] K. Burns, C. Pugh, M. Shub and A. Wilkinson, *Recent results about stable ergodicity*, in Smooth Ergodic Theory and Its Applications, edited by A. Katok, R. de la Llave, Ya. Pesin and H. Weiss, Proc. Symp. Pure Math. 69, Amer. Math. Soc., 2001, pp. 327–366.
- [46] K. Burns and R. Spatzier, *Manifolds of nonpositive curvature and their buildings*, Inst. Hautes Études Sci. Publ. Math. **65** (1987), 35–59.
- [47] K. Burns and A. Wilkinson, *Stable ergodicity of skew products*, Ann. Sci. École Norm. Sup. (4) **32** (1999), 859–889.
- [48] K. Burns and A. Wilkinson, *On the ergodicity of partially hyperbolic systems*, Ann. of Math. (2), to appear.
- [49] D. Bylov, R. Vinograd, D. Grobman and V. Nemyckii, *Theory of Lyapunov exponents and its application to problems of stability*, Izdat. “Nauka”, Moscow, 1966, in Russian.
- [50] M. Chaperon and F. Coudray, *Invariant manifolds, conjugacies and blow-up*, Ergodic Theory Dynam. Systems **17** (1997), 783–791.
- [51] B. Chirikov, *A universal instability of many-dimensional oscillator systems*, Phys. Rep. **52** (1979), 264–379.
- [52] I. Cornfeld, S. Fomin and Ya. Sinai, *Ergodic Theory*, Springer, 1982.
- [53] R. de la Llave and C. Wayne, *On Irwin’s proof of the pseudostable manifold theorem*, Math. Z. **219** (1995), 301–321.
- [54] D. Dolgopyat, *On dynamics of mostly contracting diffeomorphisms*, Comm. Math. Phys. **213** (2000), 181–201.
- [55] D. Dolgopyat, *Limit theorems for partially hyperbolic systems*, Trans. Amer. Math. Soc. **356** (2004), 1637–1689.
- [56] D. Dolgopyat, *On differentiability of SRB states for partially hyperbolic systems*, Invent. Math. **155** (2004), 389–449.
- [57] D. Dolgopyat, *Averaging and invariant measures*, Moscow Math. J. **5** (2005), 55–66.
- [58] D. Dolgopyat, H. Hu and Ya. Pesin, *An example of a smooth hyperbolic measure with countably many ergodic components*, in Smooth Ergodic Theory and Its Applications, edited by A. Katok, R. de la Llave, Ya. Pesin and H. Weiss, Proc. Symp. Pure Math. 69, Amer. Math. Soc., 2001, pp. 102–115.

- [59] D. Dolgopyat and Ya. Pesin, *Every compact manifold carries a completely hyperbolic diffeomorphism*, Ergodic Theory Dynam. Systems **22** (2002), 409–435.
- [60] D. Dolgopyat and A. Wilkinson, *Stable accessibility is C^1 dense*, in Geometric methods in dynamics. II, Astérisque 287, 2003, pp. 33–60.
- [61] P. Eberlein, *Geodesic flows on certain manifolds without conjugate points*, Trans. Amer. Math. Soc. **167** (1972), 151–170.
- [62] P. Eberlein, *Geodesic flows on negatively curved manifolds. I*, Ann. of Math. (2) **95** (1972), 492–510.
- [63] P. Eberlein, *When is a geodesic flow of Anosov type? I*, J. Differential Geometry **8** (1973), 437–463.
- [64] P. Eberlein, *When is a geodesic flow of Anosov type? II*, J. Differential Geometry **8** (1973), 565–577.
- [65] J. Eckmann and D. Ruelle, *Ergodic theory of chaos and strange attractors*, Rev. Modern Phys. **57** (1985), 617–656.
- [66] L. Eliasson, *Reducibility and point spectrum for linear quasi-periodic skew-products*, Proceedings of the International Congress of Mathematicians (Berlin, 1998), Doc. Math. **Extra Vol. II** (1998), 779–787.
- [67] D. Epstein, *Foliations with all leaves compact*, Ann. Inst. Fourier (Grenoble) **26** (1976), 265–282.
- [68] A. Fathi, M. Herman and J. Yoccoz, *A proof of Pesin’s stable manifold theorem*, in Geometric Dynamics (Rio de Janeiro, 1981), edited by J. Palis, Lect. Notes. in Math. 1007, Springer, 1983, pp. 177–215.
- [69] A. Fathi, F. Laudenbach and V. Poénaru, *Travaux de Thurston sur les surfaces*, in Séminaire Orsay, Astérisque 66–67, 1979.
- [70] N. Fenichel, *Persistence and smoothness of invariant manifolds for flows*, Indiana Univ. Math. J. **21** (1971/1972), 193–226.
- [71] R. Feres and A. Katok, *Ergodic theory and dynamics of G -spaces (with special emphasis on rigidity phenomena)*, in Handbook of Dynamical Systems 1A, edited by B. Hasselblatt and A. Katok, Elsevier, 2002, pp. 665–763.
- [72] G. Forni, *On the Lyapunov exponents of the Kontsevich–Zorich cocycle*, in Handbook of Dynamical Systems 1B, edited by B. Hasselblatt and A. Katok, Elsevier, 2006, pp. 549–580.
- [73] A. Freire and R. Mañé, *On the entropy of the geodesic flow in manifolds without conjugate points*, Invent. Math. **69** (1982), 375–392.
- [74] M. Gerber, *Conditional stability and real analytic pseudo-Anosov maps*, Mem. Amer. Math. Soc. **54** (1985), no. 321.
- [75] M. Gerber and A. Katok, *Smooth models of Thurston’s pseudo-Anosov maps*, Ann. Sci. École Norm. Sup. (4) **15** (1982), 173–204.
- [76] I. Gol’dsheid and G. Margulis, *Lyapunov indices of a product of random matrices*, Russian Math. Surveys **44** (1989), 11–71.
- [77] R. Gunesch, *Precise asymptotics for periodic orbits of the geodesic flow in nonpositive curvature*, preprint, 2003.
- [78] R. Gunesch, *Precise volume estimates in nonpositive curvature*, preprint, 2003.
- [79] J. Hadamard, *Sur l’itération et les solutions asymptotiques des équations différentielles*, Bull. Soc. Math. France **29** (1901), 224–228.
- [80] H. Hasselblatt, *Regularity of the Anosov splitting and of horospheric foliations*, Ergodic Theory Dynam. Systems **14** (1994), 645–666.
- [81] B. Hasselblatt and Ya. Pesin, *Partially hyperbolic dynamical systems*, in Handbook of Dynamical Systems 1B, edited by B. Hasselblatt and A. Katok, Elsevier, 2006, pp. 1–55.
- [82] M. Hénon, *A two dimensional mapping with a strange attractor*, Comm. Math. Phys. **50** (1976), 69–77.
- [83] M. Herman, *Une méthode pour minorer les exposants de Lyapunov et quelques exemples montrant le caractère local d’un théorème d’Arnold et de Moser sur le tore de dimension 2*, Comment. Math. Helv. **58** (1983), 453–502.
- [84] M. Hirayama, *Periodic probability measures are dense in the set of invariant measures*, Discrete Contin. Dyn. Syst. **9** (2003), 1185–1192.
- [85] M. Hirayama and Ya. Pesin, *Non-absolutely continuous invariant foliations*, Israel J. Math., to appear.
- [86] M. Hirsch, C. Pugh and M. Shub, *Invariant Manifolds*, Lect. Notes. in Math. 583, Springer, 1977.
- [87] E. Hopf, *Statistik der geodätischen Linien in mannigfaltigkeiten negativer krümmung*, Ber. Verh. Sächs. Akad. Wiss. Leipzig **91** (1939), 261–304.
- [88] H. Hu, *Some ergodic properties of commuting diffeomorphisms*, Ergodic Theory Dynam. Systems **13** (1993), 73–100.
- [89] H. Hu, Ya. Pesin and A. Talitskaya, *Every compact manifold carries a hyperbolic Bernoulli flow*, in Modern Dynamical Systems and Applications, Cambridge Univ. Press, 2004, pp. 347–358.
- [90] H. Hu, Ya. Pesin and A. Talitskaya, *Coexistence of zero and nonzero Lyapunov exponents on positive measure subsets in a transitive volume-preserving diffeomorphism*, preprint, 2006.
- [91] S. Hurder and A. Katok, *Ergodic theory and Weil measures for foliations*, Ann. of Math. (2) **126** (1987), 221–275.
- [92] M. Irwin, *A new proof of the pseudostable manifold theorem*, J. London Math. Soc. (2) **21** (1980), 557–566.
- [93] M. Jakobson, *Absolutely continuous invariant measures for one-parameter families of one-dimensional maps*, Comm. Math. Phys. **81** (1981), 39–88.

- [94] M. Jakobson and G. Świątek, *One-dimensional maps*, in Handbook of Dynamical Systems 1A, edited by B. Hasselblatt and A. Katok, Elsevier, 2002, pp. 599–664.
- [95] B. Kalinin and V. Sadovskaya, *On pointwise dimension of non-hyperbolic measures*, Ergodic Theory Dynam. Systems **22** (2002), 1783–1801.
- [96] R. Johnson, *On a Floquet theory for almost periodic, two-dimensional linear systems*, J. Differential Equations **37** (1980), 184–205.
- [97] V. Kaĭmanovich, *Lyapunov exponents, symmetric spaces and a multiplicative ergodic theorem for semisimple Lie groups*, J. Soviet Math. **47** (1989), 2387–2398.
- [98] A. Karlsson and G. Margulis, *A multiplicative ergodic theorem and nonpositively curved spaces*, Comm. Math. Phys. **208** (1999), 107–123.
- [99] A. Katok, *A conjecture about entropy*, in Smooth Dynamical Systems, edited by D. Anosov, Mir, 1977, pp. 181–203.
- [100] A. Katok, *Bernoulli diffeomorphisms on surfaces*, Ann. of Math. (2) **110** (1979), 529–547.
- [101] A. Katok, *Lyapunov exponents, entropy and periodic orbits for diffeomorphisms*, Inst. Hautes Études Sci. Publ. Math. **51** (1980), 137–173.
- [102] A. Katok, *Nonuniform hyperbolicity and structure of smooth dynamical systems*, in Proceedings of the International Congress of Mathematicians (Warsaw, 1983), PWN, 1984, pp. 1245–1253.
- [103] A. Katok, with the collaboration of K. Burns, *Infinitesimal Lyapunov functions, invariant cone families and stochastic properties of smooth dynamical systems*, Ergodic Theory Dynam. Systems **14** (1994), 757–785.
- [104] A. Katok and B. Hasselblatt, *Introduction to the Modern Theory of Dynamical Systems*, Cambridge Univ. Press, 1995.
- [105] A. Katok and J. Lewis, *Global rigidity results for lattice actions on tori and new examples of volume-preserving action*, Israel J. Math. **93** (1996), 253–280.
- [106] A. Katok and L. Mendoza, *Dynamical systems with nonuniformly hyperbolic behavior*, supplement to Introduction to the Modern Theory of Dynamical Systems, by A. Katok and B. Hasselblatt, Cambridge Univ. Press, 1995.
- [107] A. Katok and L. Mendoza, *Smooth ergodic theory*, unpublished notes.
- [108] A. Katok and R. Spatzier, *First cohomology of Anosov actions of higher rank Abelian groups and applications to rigidity*, Inst. Hautes Études Sci. Publ. Math. **79** (1994), 131–156.
- [109] A. Katok and R. Spatzier, *Subelliptic estimates of polynomial differential operators and applications to cocycle rigidity*, Math. Research Letters **1** (1994), 193–202.
- [110] A. Katok and J.-M. Strelcyn, with the collaboration of F. Ledrappier and F. Przytycki, *Invariant Manifolds, Entropy and Billiards: Smooth Maps with Singularities*, Lect. Notes in Math. 1222, Springer, 1986.
- [111] S. Kerckhoff, H. Masur and J. Smillie, *Ergodicity of billiard flows and quadratic differentials*, Ann. of Math. (2) **124** (1986), 293–311.
- [112] Yu. Kifer, *Ergodic Theory of Random Transformations*, Progress in Probability and Statistics 10, Birkhäuser, 1986.
- [113] Yu. Kifer and P.-D. Liu, *Random dynamics*, in Handbook of Dynamical Systems 1B, edited by B. Hasselblatt and A. Katok, Elsevier, 2006, pp. 379–499.
- [114] J. Kingman, *Subadditive processes*, in École d’Été de Probabilités de Saint-Flour V–1975, Lect. Notes in Math. 539, Springer, 1976, pp. 167–223.
- [115] G. Knieper, *Hyperbolic dynamics and Riemannian geometry*, in Handbook of Dynamical Systems 1A, edited by B. Hasselblatt and A. Katok, Elsevier, 2002, pp. 453–545.
- [116] M. Kontsevich, *Lyapunov exponents and Hodge theory*, in The Mathematical Beauty of Physics (Saclay, 1996), Adv. Ser. Math. Phys. 24, World Sci. Publ., 1997, pp. 318–332.
- [117] A. Krámli, N. Simányi and D. Szász, *A “transversal” fundamental theorem for semi-dispersing billiards*, Comm. Math. Phys. **129** (1990), 535–560; errata in **138** (1991), 207–208.
- [118] U. Krengel, *On Rudolph’s representation of aperiodic flows*, Ann. Inst. H. Poincaré Sect. B (N.S.) **12** (1976), 319–338.
- [119] R. Krikorian, *Réductibilité des systèmes produits-croisés à valeurs dans des groupes compacts*, Astérisque 259, 1999.
- [120] F. Ledrappier, *Propriétés ergodiques des mesures de Sinai*, Inst. Hautes Études Sci. Publ. Math. **59** (1984), 163–188.
- [121] F. Ledrappier, *Dimension of invariant measures*, in Proceedings of the Conference on Ergodic Theory and Related Topics, II (Georgenthal, 1986), Teubner-Texte Math. 94, Teubner, 1987, pp. 116–124.
- [122] F. Ledrappier and M. Misiurewicz, *Dimension of invariant measures for maps with exponent zero*, Ergodic Theory Dynam. Systems **5** (1985), 595–610.
- [123] F. Ledrappier and J.-M. Strelcyn, *A proof of the estimate from below in Pesin’s entropy formula*, Ergodic Theory Dynam. Systems **2** (1982), 203–219.
- [124] F. Ledrappier and L.-S. Young, *The metric entropy of diffeomorphisms. I. Characterization of measures satisfying Pesin’s entropy formula*, Ann. of Math. (2) **122** (1985), 509–539.
- [125] F. Ledrappier and L.-S. Young, *The metric entropy of diffeomorphisms. II. Relations between entropy, exponents and dimension*, Ann. of Math. (2) **122** (1985), 540–574.
- [126] J. Lewowicz, *Lyapunov functions and topological stability*, J. Differential Equations **38** (1980), 192–209.

- [127] J. Lewowicz, *Lyapunov functions and stability of geodesic flows*, in Geometric Dynamics (Rio de Janeiro, 1981), edited by J. Palis, Lect. Notes in Math. 1007, Springer, 1983, pp. 463–479.
- [128] J. Lewowicz and E. Lima de Sá, *Analytic models of pseudo-Anosov maps*, Ergodic Theory Dynam. Systems **6** (1986), 385–392.
- [129] P.-D. Liu and M. Qian, *Smooth Ergodic Theory of Random Dynamical Systems*, Lect. Notes in Math. 1606, Springer, 1995.
- [130] C. Liverani, *Birth of an elliptic island in a chaotic sea*, Math. Phys. Electron. J. **10** (2004), Paper 1, 13 pp.
- [131] C. Liverani and M. Wojtkowski, *Ergodicity in Hamiltonian systems*, in Dynamics Reported Expositions in Dynamical Systems (N.S.) 4, Springer, 1995, pp. 130–202.
- [132] S. Luzzatto, *Stochastic-like behaviour in nonuniformly expanding maps*, in Handbook of Dynamical Systems 1B, edited by B. Hasselblatt and A. Katok, Elsevier, 2006, pp. 265–326.
- [133] S. Luzzatto and M. Viana, *Parameter exclusions in Hénon-like systems*, Russian Math. Surveys **58** (2003), 1053–1092.
- [134] A. Lyapunov, *The General Problem of the Stability of Motion*, Taylor & Francis, 1992.
- [135] R. MacKay, *Hyperbolic structure in classical chaos*, in Quantum Chaos (Varenna, 1991), Proc. Internat. School of Phys. Enrico Fermi CXIX, North-Holland, 1993, pp. 1–50.
- [136] R. MacKay, *Renormalisation in area-preserving maps*, Advanced Series in Nonlinear Dynamics 6, World Scientific, 1993.
- [137] R. Mañé, *A proof of Pesin’s formula*, Ergodic Theory Dynam. Systems **1** (1981), 95–102, errata in **3** (1983), 159–160.
- [138] R. Mañé, *Lyapunov exponents and stable manifolds for compact transformations*, in Geometric Dynamics (Rio de Janeiro, 1981), edited by J. Palis, Lect. Notes in Math. 1007, Springer, 1983, pp. 522–577.
- [139] R. Mañé, *Ergodic Theory and Differentiable Dynamics*, Ergebnisse der Mathematik und ihrer Grenzgebiete 3. Folge-Band 8, Springer, 1987.
- [140] R. Mañé, *The Lyapunov exponents of generic area preserving diffeomorphisms*, in International Conference on Dynamical Systems (Montevideo, 1995), Pitman Res. Notes Math. Ser. 362, Longman, 1996, pp. 110–119.
- [141] R. Mañé, *On the topological entropy of geodesic flows*, J. Differential Geom. **45** (1997), 74–93.
- [142] A. Manning, *Topological entropy and the first homology group*, in Dynamical Systems (Warwick 1974), edited by A. Manning, Lect. Notes in Math. 468, Springer, 1975, pp. 185–190.
- [143] A. Manning, *Topological entropy for geodesic flows*, Ann. of Math. (2) **110** (1979), 567–573.
- [144] G. Margulis, *Certain applications of ergodic theory to the investigation of manifolds of negative curvature*, Funct. Anal. Appl. **3** (1969), 335–336.
- [145] G. Margulis, *On some Aspects of the Theory of Anosov Systems*, with a survey by R. Sharp, Springer Monographs in Mathematics, Springer, 2004.
- [146] R. Markarian, *Non-uniformly hyperbolic billiards*, Ann. Fac. Sci. Toulouse Math. (5) **3** (1994), 1207–1239.
- [147] H. Masur, *Interval exchange transformations and measured foliations*, Ann. of Math. (2) **115** (1982), 169–200.
- [148] H. Masur, *Ergodic theory of translation surfaces*, in Handbook of Dynamical Systems 1B, edited by B. Hasselblatt and A. Katok, Elsevier, 2006, pp. 527–547.
- [149] P. Mattila, *Geometry of sets and measures in Euclidean spaces*, Cambridge studies in advanced mathematics 44, Cambridge Univ. Press, 1995.
- [150] H. McCluskey and A. Manning, *Hausdorff dimension for horseshoes*, Ergodic Theory Dynam. Systems **3** (1983), 251–260; errata in **3** (1983), 319.
- [151] C. McMullen, *Dynamics on $K3$ surfaces: Salem numbers and Siegel disks*, J. Reine Angew. Math. **545** (2002), 201–233.
- [152] J. Milnor, *Fubini foiled: Katok’s paradoxical example in measure theory*, Math. Intelligencer **19** (1997), 30–32.
- [153] M. Misiurewicz and F. Przytycki, *Entropy conjecture for tori*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. **25** (1977), 575–578.
- [154] M. Misiurewicz and F. Przytycki, *Topological entropy and degree of smooth mappings*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. **25** (1977), 573–574.
- [155] C. Moore, *Amenable subgroups of semisimple groups and proximal flows*, Israel J. Math. **34** (1979), 121–138.
- [156] L. Mora and M. Viana, *Abundance of strange attractors*, Acta Math. **171** (1993), 1–71.
- [157] J. Moser, *On the volume elements on a manifold*, Trans. Amer. Math. Soc. **120** (1965), 286–294.
- [158] S. Newhouse, *Continuity properties of entropy*, Ann. of Math. (2) **129** (1989), 215–235.
- [159] Z. Nitecki, *Differentiable Dynamics. An Introduction to the Orbit Structure of Diffeomorphisms*, M.I.T. Press, 1971.
- [160] V. Nižića and A. Török, *An open dense set of stably ergodic diffeomorphisms in a neighborhood of a non-ergodic one*, Topology **40** (2001), 259–278.
- [161] D. Ornstein and B. Weiss, *Geodesic flows are Bernoullian*, Israel J. Math. **14** (1973), 184–198.
- [162] V. Oseledets, *A multiplicative ergodic theorem. Liapunov characteristic numbers for dynamical systems*, Trans. Moscow Math. Soc. **19** (1968), 197–221.

- [163] R. Osserman and P. Sarnak, *A new curvature invariant and entropy of geodesic flows*, Invent. Math. **77** (1984), 455–462.
- [164] O. Perron, *Über stabilität und asymptotisches Verhalten der Lösungen eines Systemes endlicher Differenzengleichungen*, J. Reine Angew. Math. **161** (1929), 41–64.
- [165] O. Perron, *Die stabilitätsfrage bei Differenzengleichungen*, Math. Zs. **32** (1930), 703–728.
- [166] Ya. Pesin, *An example of a nonergodic flow with nonzero characteristic exponents*, Func. Anal. and its Appl. **8** (1974), 263–264.
- [167] Ya. Pesin, *Families of invariant manifolds corresponding to nonzero characteristic exponents*, Math. USSR-Izv. **40** (1976), 1261–1305.
- [168] Ya. Pesin, *A description of the π -partition of a diffeomorphism with an invariant measure*, Math. Notes **22** (1977), 506–515.
- [169] Ya. Pesin, *Characteristic Ljapunov exponents, and smooth ergodic theory*, Russian Math. Surveys **32** (1977), 55–114.
- [170] Ya. Pesin, *Geodesic flows on closed Riemannian manifolds without focal points*, Math. USSR-Izv. **11** (1977), 1195–1228.
- [171] Ya. Pesin, *Formulas for the entropy of the geodesic flow on a compact Riemannian manifold without conjugate points*, Math. Notes **24:4** (1978), 796–805.
- [172] Ya. Pesin, *Dimension Theory in Dynamical Systems: Contemporary Views and Applications*, Chicago Lectures in Mathematics, Chicago Univ. Press, 1997.
- [173] Ya. Pesin, *General theory of smooth hyperbolic dynamical systems*, in Dynamical Systems, Ergodic Theory and Applications II, Ergodic Theory of Smooth Dynamical Systems, edited by Ya. Sinai, Encyclopedia of Mathematical Sciences 100, Springer, 2000, pp. 113–191.
- [174] Ya. Pesin, *Lectures on Partial Hyperbolicity and Stable Ergodicity*, Zürich Lectures in Advanced Mathematics, European Math. Soc., 2004.
- [175] Ya. Pesin and Ya. Sinai, *Hyperbolicity and stochasticity of dynamical systems*, Soviet Sci. Rev. Sect. C: Math. Phys. Rev. 2, Harwood Academic, 1981, pp. 53–115.
- [176] Ya. Pesin and Ya. Sinai, *Gibbs measures for partially hyperbolic attractors*, Ergodic Theory Dynam. Systems **2** (1982), 417–438.
- [177] Ya. Pesin and H. Weiss, *On the dimension of deterministic and random Cantor-like sets, symbolic dynamics, and the Eckmann–Ruelle conjecture*, Comm. Math. Phys. **182** (1996), 105–153.
- [178] K. Petersen, *Ergodic Theory*, Cambridge Studies in Advanced Mathematics 2, Cambridge Univ. Press, 1989.
- [179] M. Pollicott, *Lectures on Ergodic Theory and Pesin Theory on Compact Manifolds*, London Mathematical Society Lecture Note Series 180, Cambridge University Press, 1993.
- [180] V. Potapov, *The multiplicative structure of J -contractive matrix functions*, Amer. Math. Soc. Transl. (2) **15** (1960), 131–243.
- [181] V. Potapov, *A theorem on the modulus. I*, Amer. Math. Soc. Transl. (2) **138** (1988), 55–65.
- [182] V. Potapov, *Linear fractional transformation of matrices*, Amer. Math. Soc. Transl. (2) **138** (1988), 21–35.
- [183] F. Przytycki, *Examples of conservative diffeomorphisms of the two-dimensional torus with coexistence of elliptic and stochastic behaviour*, Ergodic Theory Dynam. Systems **2** (1982), 439–463.
- [184] C. Pugh, *The $C^{1+\alpha}$ hypothesis in Pesin theory*, Inst. Hautes Études Sci. Publ. Math. **59** (1984), 143–161.
- [185] C. Pugh and M. Shub, *Ergodic attractors*, Trans. Amer. Math. Soc. **312** (1989), 1–54.
- [186] C. Pugh and M. Shub, *Stable ergodicity and Julienne quasi-conformality*, J. European Math. Soc. **2** (2000), 1–52.
- [187] M. Ragunathan, *A proof of Oseledec’s multiplicative ergodic theorem*, Israel J. Math. **32** (1979), 356–362.
- [188] S. Rasband, *Chaotic Dynamics of Nonlinear Systems*, John Wiley & Sons, 1990.
- [189] A. Rodríguez Hertz, F. Rodríguez Hertz and R. Ures, *Partially hyperbolic systems with 1D-center bundle: I. Stable ergodicity*, preprint.
- [190] D. Rudolph, *A two-valued step coding for ergodic flows*, Math. Z. **150** (1976), 201–220.
- [191] D. Ruelle, *A measure associated with Axiom A attractors*, Amer. J. Math. **98** (1976), 619–654.
- [192] D. Ruelle, *An inequality for the entropy of differentiable maps*, Bol. Soc. Brasil. Mat. **9** (1978), 83–87.
- [193] D. Ruelle, *Ergodic theory of differentiable dynamical systems*, Inst. Hautes Études Sci. Publ. Math. **50** (1979), 27–58.
- [194] D. Ruelle, *Characteristic exponents and invariant manifolds in Hilbert space*, Ann. of Math. (2) **115** (1982), 243–290.
- [195] D. Ruelle, *Perturbation theory for Lyapunov exponents of a toral map: extension of a result of Shub and Wilkinson*, Israel J. Math. **134** (2003), 345–361.
- [196] D. Ruelle and A. Wilkinson, *Absolutely singular dynamical foliations*, Comm. Math. Phys. **219** (2001), 481–487.
- [197] R. Sacker and G. Sell, *Lifting properties in skew-product flows with applications to differential equations*, Mem. Amer. Math. Soc. **11** (1977), no. 190.
- [198] R. Sacker and G. Sell, *A spectral theory for linear differential systems*, J. Differential Equations **27** (1978), 320–358.

- [199] G. Sell, *The structure of a flow in the vicinity of an almost periodic motion*, J. Differential Equations **27** (1978), 359–393.
- [200] M. Shub and A. Wilkinson, *Pathological foliations and removable zero exponents*, Invent. Math. **139** (2000), 495–508.
- [201] Ya. Sinai, *Classical dynamic systems with countably-multiple Lebesgue spectrum. II*, Izv. Akad. Nauk SSSR Ser. Mat **30** (1966), 15–68.
- [202] Ya. Sinai, *Dynamical systems with elastic reflections*, Russ. Math. Surveys **68** (1970), 137–189.
- [203] Ya. Sinai, *Some rigorous results on decay of correlations*, supplement to the book *Statistical Irreversibility in Nonlinear Systems* by G. Zaslavskij, Nauka, 1970, pp. 124–139.
- [204] Ya. Sinai, *Gibbs measures in ergodic theory*, Russian Math. Surveys **27** (1972), 21–69.
- [205] Ya. Sinai and N. Chernov, *Entropy of hard spheres gas with respect to the group of space-time translations*, Tr. Semin. Im. I. G. Petrovskogo **8** (1982), 218–238, in Russian.
- [206] S. Smale, *Diffeomorphisms of the 2-sphere*, Proc. Amer. Math. Soc. **10** (1959), 621–626.
- [207] S. Smale, *Diffeomorphisms with many periodic points*, in Differential and Combinatorial Topology (A symposium in Honor of Marston Morse), Princeton Univ. Press, 1965, pp. 63–80.
- [208] S. Smale, *Differentiable dynamical systems*, Bull. Amer. Math. Soc. (N.S.) **73** (1967), 747–817.
- [209] A. Tahzibi, *C^1 -generic Pesin's entropy formula*, C. R. Math. Acad. Sci. Paris **335** (2002), 1057–1062.
- [210] A. Talitskaya, *Partially Hyperbolic Phenomena in Dynamical Systems with Discrete and Continuous Time*, Ph.D. Thesis, Pennsylvania State University, 2004.
- [211] Ph. Thieullen, *Fibrés dynamiques asymptotiquement compacts. Exposants de Lyapunov. Entropie. Dimension*, Ann. Inst. H. Poincaré. Anal. Non Linéaire **4** (1987), 49–97.
- [212] Ph. Thieullen, *Ergodic reduction of random products of two-by-two matrices*, J. Anal. Math. **73** (1997), 19–64.
- [213] W. Thurston, *On the geometry and dynamics of diffeomorphisms of surfaces*, Bull. Amer. Math. Soc. (N.S.) **19** (1988), 417–431.
- [214] M. Tsujii, *Regular points for ergodic Sinai measures*, Trans. Amer. Math. Soc. **328** (1991), 747–777.
- [215] I. Ugarcovici, *On hyperbolic measures and periodic orbits*, Discrete Contin. Dyn. Syst. **16** (2006), 505–512.
- [216] W. Veech, *Gauss measures for transformations on the space of interval exchange maps*, Ann. of Math. (2) **115** (1982), 201–242.
- [217] W. Veech, *The Teichmüller geodesic flow*, Ann. of Math. (2) **124** (1986), 441–530.
- [218] W. Veech, *Flat surfaces*, Amer. J. Math. **115** (1993), 589–689.
- [219] M. Viana, *Strange attractors in higher dimensions*, Bol. Soc. Brasil. Mat. (N.S.) **24** (1993), 13–62.
- [220] M. Viana, *Multidimensional nonhyperbolic attractors*, Inst. Hautes Études Sci. Publ. Math. **85** (1997), 63–96.
- [221] M. Viana, *Almost all cocycles over any hyperbolic system have non-vanishing Lyapunov exponents*, Ann. of Math. (2), to appear.
- [222] Q. Wang and L.-S. Young, *Strange attractors with one direction of instability*, Comm. Math. Phys. **218** (2001), 1–97.
- [223] Q. Wang and L.-S. Young, *From invariant curves to strange attractors*, Comm. Math. Phys. **225** (2002), 275–304.
- [224] Z. Wang and W. Sun, *Liapunov exponents of hyperbolic measure and hyperbolic periodic points*, preprint.
- [225] M. Wojtkowski, *Invariant families of cones and Lyapunov exponents*, Ergodic Theory Dynam. Systems **5** (1985), 145–161.
- [226] M. Wojtkowski, *Measure theoretic entropy of the system of hard spheres*, Ergodic Theory Dynam. Systems **8** (1988), 133–153.
- [227] M. Wojtkowski, *Systems of classical interacting particles with nonvanishing Lyapunov exponents*, in Lyapunov exponents (Oberwolfach, 1990), edited by L. Arnold, H. Crauel and J.-P. Eckmann, Lect. Notes in Math. 1486, Springer, 1991, pp. 243–262.
- [228] M. Wojtkowski, *W-flows on Weyl manifolds and Gaussian thermostats*, J. Math. Pures Appl. (9) **79** (2000), 953–974.
- [229] M. Wojtkowski, *Magnetic flows and Gaussian thermostats on manifolds of negative curvature*, Fund. Math. **163** (2000), 177–191.
- [230] M. Wojtkowski, *Monotonicity, J-algebra of Potapov and Lyapunov exponents*, in Smooth Ergodic Theory and Its Applications, edited by A. Katok, R. de la Llave, Ya. Pesin and H. Weiss, Proc. Symp. Pure Math. 69, Amer. Math. Soc., 2001, pp. 499–521.
- [231] M. Wojtkowski and C. Liverani, *Conformally symplectic dynamics and symmetry of the Lyapunov spectrum*, Comm. Math. Phys. **194** (1998), 47–60.
- [232] Z. Xia, *Existence of invariant tori in volume-preserving diffeomorphisms*, Ergodic Theory Dynam. Systems **12** (1992), 621–631.
- [233] J.-C. Yoccoz, *Travaux de Herman sur les tores invariants*, Séminaire Bourbaki, Vol. 1991/92, Astérisque 206, 1992, Exp. No. 754, 4, 311–344.
- [234] Y. Yomdin, *C^k -resolution of semialgebraic mappings. Addendum to: "Volume growth and entropy"*, Israel J. Math. **57** (1987), 301–317.

- [235] Y. Yomdin, *Volume growth and entropy*, Israel J. Math. **57** (1987), 285–300.
- [236] L.-S. Young, *Dimension, entropy and Lyapunov exponents*, Ergodic Theory Dynam. Systems **2** (1982), 109–124.
- [237] L.-S. Young, *Lyapunov exponents for some quasi-periodic cocycles*, Ergodic Theory Dynam. Systems **17** (1997), 483–504.
- [238] L.-S. Young, *Statistical properties of dynamical systems with some hyperbolicity*, Ann. of Math. (2) **147** (1998), 585–650.
- [239] R. Zimmer, *Ergodic theory, group representations, and rigidity*, Bull. Amer. Math. Soc. (N.S.) **6** (1982), 383–416.
- [240] R. Zimmer, *Ergodic Theory and Semisimple Groups*, Monographs in Mathematics 81, Birkhäuser, 1984.
- [241] A. Zorich, *Finite Gauss measure on the space of interval exchange transformations. Lyapunov exponents*, Ann. Inst. Fourier (Grenoble) **46** (1996), 325–370.
- [242] A. Zorich, *How do the leaves of a closed 1-form wind around a surface?*, in Pseudoperiodic Topology, edited by V. Arnold, M. Kontsevich and A. Zorich, Amer. Math. Soc. Transl. Ser. 2, 197, Amer. Math. Soc., 1999, pp. 135–178.
- [243] A. Zorich, *Flat surfaces*, in Frontiers in Number Theory, Physics and Geometry I: On Random Matrices, Zeta Functions and Dynamical Systems, edited by P. Cartier, B. Julia, P. Moussa and P. Vanhove, Springer, 2006, pp. 439–586.

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