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*Corporate Bond and Commercial
Loan Portfolio Analysis*

by
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96-41

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“Corporate Bond and Commercial Loan Portfolio Analysis” ¹

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Abstract: In this paper we have presented a new approach to measure the return-risk trade-off in portfolios of risky debt instruments, whether bonds or loans. The use of complex, statistically based portfolio techniques to manage assets of financial institutions and fixed income portfolio money managers is very much in its early phase and will continue to evolve, perhaps more quickly in the near future. Our approach substitutes the concept of unexpected loss for the more traditional variance of return measure used in equity securities analysis. Preliminary empirical tests indicate some reason to be optimistic about this approach.

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Introduction

Increasingly financial institutions (FI), primarily banks, have recognized the need to measure credit concentration risk as well as the credit risk on individual loans. The same can be said for concentration concerns of bond portfolio managers but the urgency is less evident. The early approaches to concentration risk analysis were based either on: (1) subjective analysis (the expert's feel as to a maximum percent of loans to allocate to an economic sector or geographic location, e.g., an SIC code or Latin America) or (2) limiting exposure to a certain percent of capital in particular industries or credit rating classes. In a relatively early study, Bennett (1984) presented rating migration of bank assets in a pioneering portfolio risk discussion. He emphasized the need for a common risk rating system for all bank assets, including corporate, country, consumer loans and loans to other banks.

More recently, the potential for applying modern portfolio theory (MPT) to loans and other fixed income instruments has been recognized. One attempt at applying MPT was that of Chirinko and Guill (1991). Their approach required the use of a macro econometric model of the U.S. economy to generate future possible states of the world and thus SIC sector loan payoffs (loss rates). From the distribution of such loss rates, means, variances and covariances could be calculated and an efficient loan portfolio constructed (defined at the level of SIC code aggregation).

In the remainder of this paper, we discuss an alternative

portfolio theory based approach to analyzing the optimal composition of fixed income (either bond or loan) portfolios and present some preliminary empirical tests of this method.

Fixed Income Portfolio Analysis

Since the pioneering work of Markowitz (1959), portfolio theory has been applied to common stocks. The traditional objectives of maximizing returns for given levels of risk or minimizing risk for given levels of return have guided efforts to achieve effective diversification of portfolios. Such concepts as individual stock and portfolio betas to indicate risk levels and to calculate efficient frontiers, with optimal weighings of the portfolio's member stocks, are now common parlance among investment professionals and in textbooks, e.g., Elton and Gruber (1995). This is not to say that these concepts are widely used to the exclusion of more traditional industrial sector, geographical location, size, or some other diversification strategy. The necessary data in terms of historical returns and correlations of returns between individual stocks are usually available to perform the portfolio optimization analysis.

One might expect that these very same techniques would (and could) be applied to the fixed income area involving corporate and government bonds and even to bank loans. There has been,

however, very little published work in the bond area¹ and a recent survey of practices by commercial banks found fragmented and untested efforts.² The objective of effective risk reducing methods is, however, a major pre-occupation of financial institutions, with bank loan research departments and regulators spending considerable resources to reduce the likelihood of major loan losses that jeopardize the very existence of the lending institution. Recent bank failures attributed to huge loan losses in the United States, Japan, Europe, Latin America and Australia have raised the level of concern. Still, conceptually sound diversification techniques have eluded most bank and bond portfolio managers, probably for valid reasons.

It is the objective of this section of our paper to outline a method that will avoid the major data and analytical pitfalls that have plagued fixed income portfolio efforts and to provide a sound and empirically feasible portfolio approach. Our empirical examples will involve corporate bonds but we feel confident that the methodology is applicable as well to commercial and industrial loans.

¹Platt and Platt (1991) did some preliminary work for high yield "junk bond" portfolios by introducing a linear programming algorithm which maximized yield-to-maturity subject to a constraint as to the level of default risk and the degree of diversification. To our knowledge, however, corporate bond portfolio managers have not utilized this concept and continue to invest based on traditional industry, size, and credit rating criteria.

²McAllister and Mingo's (1994) survey concluded that commercial banks were experimenting with a number of different techniques but few had been implemented or had impacted corporate lending practices.

Return-Risk Framework

The classic mean variance of return framework is not valid for long-term, fixed income portfolio strategies. As we will show, the problem does not lie in the expected return measure on individual assets, but in the distribution of possible returns. While the fixed income investor can lose all or most of the investment in the event of default, positive returns are limited. This problem is mitigated when the measurement period of returns is relatively short, e.g., monthly, and the likely variance of returns is small and more normal. We will return to measures of portfolio risk both for short term returns and the more challenging buy-and-hold, long-term strategy.

Return Measurement

The measurement of expected portfolio return is actually quite straight-forward for fixed income bond and loan assets. The investor (or FI) is promised a fixed return (yield-to-maturity) over time and should subtract, from this promised yield, the expected losses from default of the issuer. For certain measurement periods, the return will also be influenced by changes in interest rates but we will assume, for purposes of exposition, that these changes are random with an expected capital gain of zero. Likewise, we acknowledge that investors can infer capital gains or losses from the yield curve and also from whether the bonds are trading at a premium or discount from

par.

The expected annual return, given in equation (1), is therefore:

$$\text{EAR} = \text{YTM} - \text{EAL} \quad (1)$$

where:

EAR = Expected annual return

YTM = Yield-to-Maturity (or Yield-to-Worst)

EAL = Expected Annual Loss

We derive the EAL from prior work on bond mortality rates and losses (Altman, 1988, 1989). Each bond is analyzed based on its initial (or existing)³ bond rating which implies an expected rate of default for up to ten (or longer) years after issuance. Exhibits 1 and 2 list cumulative mortality rates and cumulative mortality losses, respectively, covering the period 1971-1994.⁴ Exhibit 3 annualizes these mortality rates and losses. So, for example, a 10-year BB (S&P rated) bond has an expected annual loss of 91 basis points per year. If the newly issued BB rated bond has a promised yield of 9.0% with a spread of 2.0% over 7.0% risk-free U.S. Treasury bonds, then the expected return is 8.09% per year, or a risk premium of 109 basis points over the risk-free rate. If our measurement period were quarterly returns instead of annual, then the expected return would be about 2.025% per quarter. Again, our expected return measure is focused primarily on credit risk changes and not on yield curve

³The measurement of expected defaults for existing bonds compared to newly issued ones is essentially the same for bonds with maturities of at least five years. Moody's and S&P publish data on existing baskets of bonds by rating without regard to age. Their results and ours essentially converge after year four (see Altman, 1992).

⁴For updated data through 1995, see Altman and Kishore (1996).

EXHIBIT 1

MORTALITY RATES BY ORIGINAL RATING: ALL RATINGS OF CORPORATE BONDS*
1971-1994

Years After Issuance

| Rating | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|--------|------------|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| AA/ | Yearly | 0.00% | 0.00% | 0.00% | 0.00% | 0.08% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% |
| | Cumulative | 0.00% | 0.00% | 0.00% | 0.00% | 0.08% | 0.08% | 0.08% | 0.08% | 0.08% | 0.08% |
| AA | Yearly | 0.00% | 0.05% | 1.06% | 0.09% | 0.00% | 0.00% | 0.01% | 0.00% | 0.06% | 0.04% |
| | Cumulative | 0.00% | 0.05% | 1.11% | 1.20% | 1.20% | 1.20% | 1.20% | 1.20% | 1.26% | 1.30% |
| A | Yearly | 0.00% | 0.19% | 0.07% | 0.21% | 0.06% | 0.06% | 0.20% | 0.19% | 0.00% | 0.00% |
| | Cumulative | 0.00% | 0.19% | 0.26% | 0.47% | 0.53% | 0.59% | 0.78% | 0.98% | 0.98% | 0.98% |
| BBB | Yearly | 0.41% | 0.25% | 0.32% | 0.55% | 0.89% | 0.39% | 0.09% | 0.00% | 0.59% | 0.23% |
| | Cumulative | 0.41% | 0.66% | 0.97% | 1.51% | 2.39% | 2.77% | 2.86% | 2.86% | 3.44% | 3.66% |
| BB | Yearly | 0.50% | 0.58% | 4.15% | 4.84% | 1.13% | 0.33% | 0.94% | 0.23% | 0.64% | 0.58% |
| | Cumulative | 0.50% | 1.08% | 5.19% | 9.78% | 10.79% | 11.26% | 13.64% | 13.87% | 14.55% | 15.21% |
| B | Yearly | 1.59% | 7.12% | 6.80% | 7.29% | 3.40% | 3.40% | 2.80% | 2.13% | 2.83% | 3.43% |
| | Cumulative | 1.59% | 8.60% | 14.82% | 21.02% | 23.71% | 28.21% | 30.22% | 31.70% | 33.63% | 35.91% |
| CCC | Yearly | 8.32% | 10.69% | 18.53% | 10.26% | 9.18% | 5.56% | 2.49% | 2.97% | 12.28% | 1.35% |
| | Cumulative | 8.32% | 18.13% | 33.30% | 40.14% | 45.63% | 48.66% | 49.94% | 51.42% | 57.39% | 58.31% |

*Rated by S&P at issuance

Source: E. Altman and V. Kishore (1995)

EXHIBIT 2

MORTALITY LOSSES BY ORIGINAL RATING ALL RATINGS OF CORPORATE BONDS*
1971-1994

Years After Issuance

| Rating | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|--------|------------|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| AAA | Yearly | 0.00% | 0.00% | 0.00% | 0.00% | 0.08% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% |
| | Cumulative | 0.00% | 0.00% | 0.00% | 0.00% | 0.08% | 0.08% | 0.08% | 0.08% | 0.08% | 0.08% |
| AA | Yearly | 0.00% | 0.02% | 0.21% | 0.03% | 0.00% | 0.00% | 0.01% | 0.00% | 0.04% | 0.02% |
| | Cumulative | 0.00% | 0.02% | 0.23% | 0.26% | 0.26% | 0.26% | 0.26% | 0.26% | 0.30% | 0.32% |
| A | Yearly | 0.00% | 0.03% | 0.02% | 0.15% | 0.06% | 0.03% | 0.11% | 0.13% | 0.00% | 0.00% |
| | Cumulative | 0.00% | 0.03% | 0.05% | 0.20% | 0.26% | 0.29% | 0.40% | 0.52% | 0.52% | 0.52% |
| BBB | Yearly | 0.27% | 0.10% | 0.21% | 0.26% | 0.36% | 0.30% | 0.06% | 0.00% | 0.41% | 0.14% |
| | Cumulative | 0.27% | 0.37% | 0.58% | 0.84% | 1.19% | 1.49% | 1.55% | 1.55% | 1.95% | 2.08% |
| BB | Yearly | 0.26% | 0.26% | 3.34% | 2.14% | 0.70% | 0.33% | 0.94% | 0.23% | 0.64% | 0.58% |
| | Cumulative | 0.26% | 0.51% | 3.84% | 5.90% | 6.56% | 6.86% | 7.74% | 7.95% | 8.54% | 9.07% |
| B | Yearly | 0.83% | 5.12% | 5.02% | 5.95% | 2.44% | 3.93% | 2.06% | 1.64% | 1.98% | 1.59% |
| | Cumulative | 0.83% | 5.90% | 10.63% | 15.95% | 18.00% | 21.22% | 22.84% | 24.11% | 25.61% | 26.79% |
| CCC | Yearly | 7.22% | 8.87% | 15.30% | 6.82% | 6.76% | 3.29% | 2.49% | 0.91% | 8.35% | 1.25% |
| | Cumulative | 7.22% | 15.45% | 28.39% | 33.27% | 37.78% | 39.83% | 41.33% | 41.87% | 47.47% | 47.61% |

*Rated by S&P at issuance

Source: E. Altman and V. Kishore (1995)

Exhibit 3

Annualized Cumulative Default Rates and Annualized Cumulative Mortality Loss Rates
(1971-1994)

| Original Rating/Year | Annualized Cumulative Default Rates | | | | | | | | | |
|----------------------|-------------------------------------|--------|--------|--------|--------|--------|--------|--------|--------|---------|
| | 1(in%) | 2(in%) | 3(in%) | 4(in%) | 5(in%) | 6(in%) | 7(in%) | 8(in%) | 9(in%) | 10(in%) |
| AAA | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| AA | 0.00 | 0.00 | 0.27 | 0.27 | 0.22 | 0.19 | 0.16 | 0.14 | 0.13 | 0.12 |
| A | 0.00 | 0.05 | 0.08 | 0.11 | 0.10 | 0.09 | 0.10 | 0.11 | 0.10 | 0.09 |
| BBB | 0.04 | 0.27 | 0.26 | 0.33 | 0.37 | 0.40 | 0.44 | 0.39 | 0.35 | 0.37 |
| BB | 0.00 | 0.35 | 1.26 | 1.44 | 2.10 | 1.91 | 2.02 | 1.81 | 1.68 | 1.59 |
| B | 0.99 | 2.14 | 4.61 | 5.01 | 5.14 | 4.71 | 4.58 | 4.25 | 3.97 | 4.09 |
| CCC | 2.24 | 8.35 | 11.75 | 10.50 | 9.87 | 9.78 | 8.82 | 8.07 | 7.21 | 8.35 |

| Original Rating/Year | Annualized Cumulative Mortality Loss Rates | | | | | | | | | |
|----------------------|--------------------------------------------|--------|--------|--------|-------------|-------------|--------|--------|--------|---------|
| | 1(in%) | 2(in%) | 3(in%) | 4(in%) | 5(in%) | 6(in%) | 7(in%) | 8(in%) | 9(in%) | 10(in%) |
| AAA | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| AA | 0.00 | 0.00 | 0.05 | 0.06 | 0.05 | 0.04 | 0.04 | 0.03 | 0.03 | 0.03 |
| A | 0.00 | 0.01 | 0.01 | 0.04 | 0.05 | 0.04 | 0.05 | 0.05 | 0.05 | 0.05 |
| BBB | 0.03 | 0.15 | 0.15 | 0.20 | 0.19 | 0.20 | 0.24 | 0.22 | 0.19 | 0.21 |
| BB | 0.00 | 0.20 | 0.86 | 1.01 | 1.22 | 1.11 | 1.09 | 0.98 | 0.94 | 0.91 |
| B | 0.42 | 1.23 | 3.29 | 3.64 | 3.81 | 3.46 | 3.36 | 3.12 | 2.91 | 2.89 |
| CCC | 1.51 | 7.19 | 9.79 | 8.69 | 7.82 | 7.57 | 6.87 | 6.13 | 7.06 | 7.25 |

Source: Calculation on data from Exhibits 1 and 2

implications.

The latter is obviously more relevant to government bond portfolios.

The problem of measuring expected returns for commercial loans is a bit more complex. Since most loans do not have a risk rating attached to it by the rating agencies,⁵ the loan portfolio analyst must utilize a proxy measure. We advocate using the bank's own risk rating system as long as each of the internal ratings is linked with the public bond ratings, e.g., those used by Altman, Moody's or S&P in their cumulative default studies.

We will also show that these proxy risk measures, either from internal systems or from commercially available systems,⁶ are critical ingredients in the compilation of historical correlations of risk and return measures between assets in the portfolio. The expected portfolio return (R_p) is therefore based on each asset's expected annual return, weighted by the proportion (X_i) of each loan/bond relative to the total portfolio;

$$R_p = \sum_{i=1}^N X_i EAR_i \quad (2)$$

⁵The rating agencies will rate loans by their private placement service but the number of these ratings are relatively few.

⁶Such systems as ZETA Services (Hoboken, NJ) and KMV (San Francisco, CA) are available to assign ratings and expected defaults to all companies, whether or not they have public debt outstanding.

Portfolio Risk and Efficient Frontiers Using Returns

The classic mean return-variance portfolio framework is given in equation 3, when we utilize a short holding period, e.g., monthly or quarterly, and historical data exists for the requisite period to calculate correlation of returns among the loans/bonds.

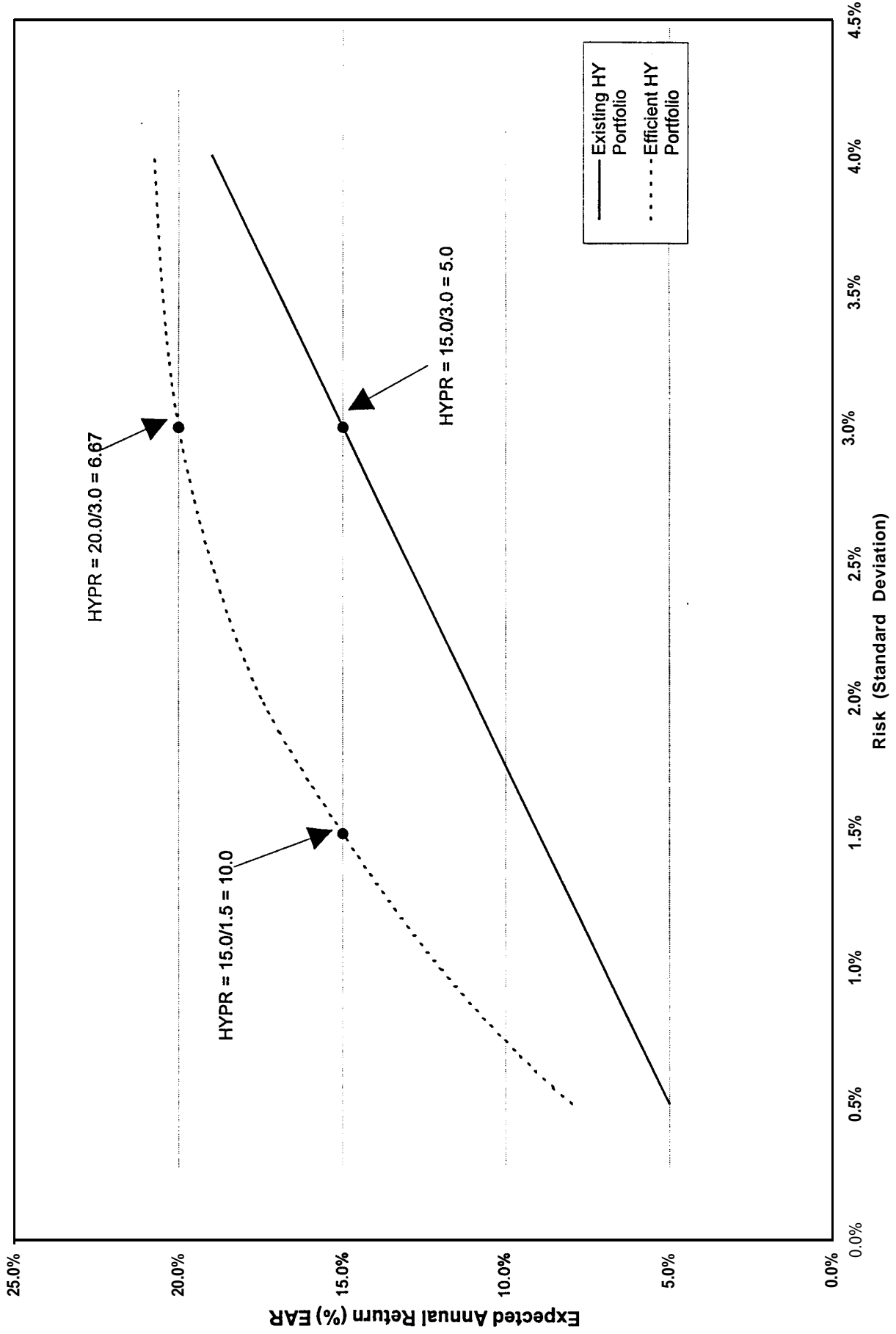
$$V_p = \sum_{i=1}^N \sum_{j=1}^N X_i X_j \sigma_i \sigma_j \rho_{ij} \quad (3)$$

where:

- V_p = Variance (Risk) of the Portfolio
- X_i = The proportion of the Portfolio Invested in Bond Issue i .
- σ_i = Standard Deviation of the Return for the Sample Period for Bond Issue i .
- ρ_{ij} = Correlation Coefficient of the Quarterly Returns for Bonds i and j .

For example, if returns on all assets exist for 60 months or 20 quarters, then the correlations are meaningful and the classic efficient frontier can be calculated. Exhibit 4 shows an efficient frontier, i.e., maximization of expected return for given levels of risk or minimization of risk (standard deviation of returns) for given levels of return, for a hypothetical high yield bond portfolio. The objective is to illustrate maximization of the HYPR (High Yield Portfolio Ratio) for given levels of risk or return. Note that an existing portfolio with a HYPR of 5.0 can be improved to 6.67 holding risk constant or to

Exhibit 4
The High Yield Portfolio Ratio (HYPR) Approach for
Risk-Return Assessment



10.0 holding return constant.

Our HYPR is a variation on the so-called Sharpe ratio, first introduced as a reward-to-variability ratio by Sharpe (1966), later popularized as the Sharpe Index or Sharpe ratio by many, e.g., Reilly (1989), Morningstar (1993), and finally generalized and expanded to cover a broader range of applications by Sharpe (1994) . Most often applied to measuring the performance of equity mutual funds, this ratio captures the average differential return (\bar{d}) between a fund (R_f) and an appropriate benchmark (R_b) and the standard deviation (σ_d) of the differences over the period. As such, it captures the average differential return per unit of risk (standard deviation), assuming the appropriate risk measure is the variance of returns.

The only other applications of a version of the Sharpe ratio to fixed income asset portfolios and derivatives were proposed in unpublished manuscripts by McQuown (1994) and Kealhofer (1996). They utilize a risk of default model developed by KMV Corporation which itself is based (indirectly) on the level, variability and correlations of the stock price of the existing and potential companies in the portfolio. Our fixed income asset portfolio model has many similarities to that of McQuown, with the major difference being the measure of default risk in the model (see a discussion of the Z and Zeta risk measures and the KMV expected default frequency approach in Altman and Saunders, (1996) .

We agree with McQuown and Kealhofer that the risk of any individual bond/loan as well as the entire portfolio itself is a

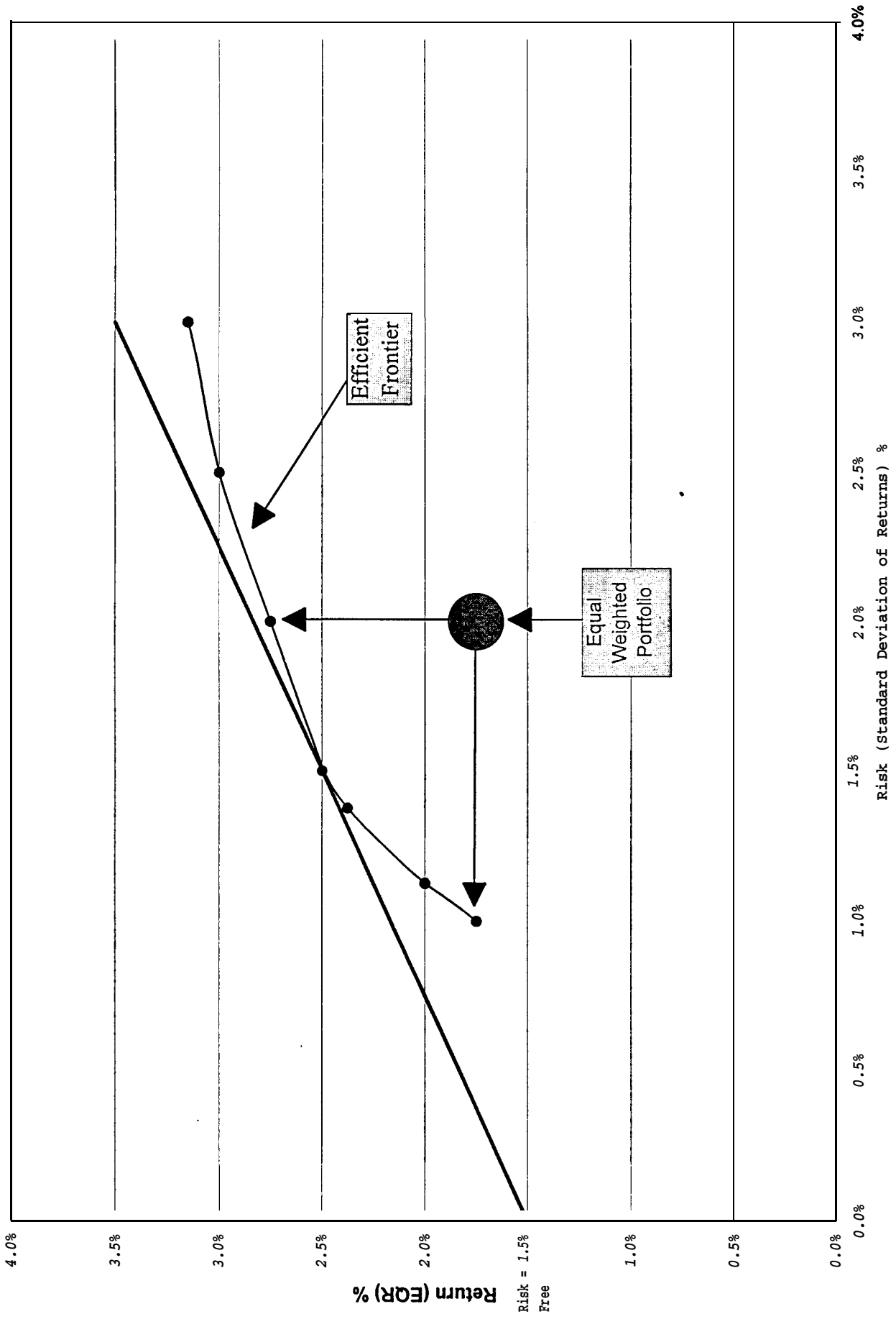
measure that incorporates the *unexpected* loss. We will return to the concept of unexpected losses shortly.

Exhibit 5 shows an efficient frontier based on a potential portfolio of 10 high yield corporate bonds utilizing actual quarterly returns from the five year period 1991-1995. The efficient portfolio compared to the equally weighted one shows considerable improvement in the return-risk tradeoff. For example, the HYPR goes from about 0.67 (2.0/3.0) to 1.14 (2.0/1.75) for the same expected return and to 1.0 (3.0/3.0) for the same variance of return. Note also the link between the risk-free rate at about 1.5% per quarter and the tangent line to the efficient frontier, indicating various proportions of risky vs. risk-free fixed income assets. The efficient frontier, calculated without any constraint as to the number of issues in the portfolio, involved eight of the possible ten high yield bonds. And, when we constrain the model such that no issue can be greater than 15% of the portfolio, the actual number of issues was either seven or eight depending upon the different expected returns, (see Exhibit 8 below).

Portfolio Risk and Efficient Frontiers Using an Alternative Risk Measure

The reality of the bond and loan markets is that even if one was comfortable with the distribution qualities of returns, the need to analyze a reasonably large number of potential assets precludes the use of the classic mean-variance of return

Exhibit 5
Efficient High Yield Bond Portfolio, Using Returns
10 Issues



framework. Specifically, there simply is insufficient historical high yield bond return and loan returns data to compute correlations. The same problem would be true if, instead of using return correlations, which can vary due to maturity differences between bonds, we utilized the correlation of the duration of each bond with other bonds and with the overall index of bonds to calculate the (i) correlation between bonds and (ii) variance of the portfolio.' Other sample selection problems include the change in maturities of individual bonds over the measurement period and the exclusion of bonds that defaulted in the past.

We analyzed the potential to use returns or durations in the high yield corporate debt market and out of almost 600 bond issues that existed as of year-end 1995, less than forty had 20 quarters of historical data. If we add to this scenario our other conceptual concerns, as indicated above, it is simply not appropriate (theoretically or empirically) to utilize the variance of return as the measure of either the individual assets' or the portfolio's risk.

An alternative risk measure, one that is critical to most bank and fixed income portfolio managers, is unexpected loss from defaults. Recall that we adjusted the promised yield for expected losses. Therefore, the risk is the downside in the

'See Elton and Gruber (1995) for an exposition on the use of the duration measure in analyzing correlation between fixed income assets.

event that the expected losses underestimate actual losses.⁸ In addition, unexpected losses are the cornerstone measure in the determination of appropriate reserves against bank capital in the RAROC (risk adjusted return on capital) approach adopted by many banks.

Our suggested approach for determining unexpected losses is to utilize a variation of the Z-Score model, called the Z"-Score model (Altman, 1993) to assign a bond rating equivalent to each of the loans/bonds that could possibly enter the portfolio.⁹ As noted earlier, these scores and rating equivalents can then be used to estimate expected losses over time. If we then observe the standard deviation around the expected losses, we have a procedure to estimate unexpected losses. For example, the expected loss on a BB rated equivalent 10 year bond is 91 basis points per year (Exhibit 3). The standard deviation around this expected value was computed to be 2.65%, or 265 basis points per year. The standard deviation is computed from the individual issuance years', independent observations that were used to calculate the cumulative mortality losses. For example, there are 24 one-year default losses, for bonds issued in a certain rating class, over the 1971-1995 period, i.e., 1971 issued bonds

⁸This idea is similar to the use of the semi-variance measure of returns, whereby the analyst is concerned only with the return below the mean.

⁹The Z"-Score model is a four variable version of the Z-Score approach. It was designed to reduce distortions in credit scores for firms in different industries. We have also found this model extremely effective in assessing the credit risk of corporate bonds in the emerging market arena, see Altman, Hartzell & Peck (1995). We call this application the EM Score approach.

defaulting in 1972, 1972 issued bonds defaulting in 1973, etc. In the same way, there are 23 two-year cumulative loss data points, 22 three-year loss observations, etc., up to 15 ten-year observations.

As noted above, the model used here is the Z"-Score (or EM Score) risk rating mode 1, indicated in equation (4) with the bond rating equivalents shown in Exhibit 6.¹⁰

$$Z''\text{-Score} = 6.56(X_1) + 3.26(X_2) + 6.72(X_3) + 1.05(X_4) + 3.25 \quad (4)$$

where: x_1 = Working Capital/Total Assets
 x_2 = Retained Earnings/Total Assets
 x_3 = EBIT/Total Assets
 x_4 = Equity (Book Value)/Total Liabilities

Portfolio Risk

The formula for our portfolio risk measure is given in equation (5).

$$UAL_p = \sum_{i=1}^N \sum_{j=1}^N X_i X_j \sigma_i \sigma_j \rho_{ij} \quad (5)$$

The measure UAL_p is the unexpected loss on the portfolio consisting of measures of individual asset unexpected losses (σ_i, σ_j) and the correlation (ρ_{ij}) of unexpected losses over the sample measurement period. Again, these unexpected losses are based on the standard deviation of annual expected losses for the

¹⁰In order to standardize our bond rating equivalent analysis, we add a constant term of 3.25 to the model; scores of zero (0) indicating a D (default) rating and positive scores indicating ratings above D. The actual bond rating equivalents are derived from a sample of over 750 U.S. corporate bonds with average scores for each rating category (shown in Exhibit 6).

Exhibit 6

U.S. Bond Rating Equivalent, Based on Z" Score

| U.S. Equivalent Rating | Average Z" Score | Sample Size |
|------------------------|------------------|-------------|
| AAA | 8.15 | 8 |
| AA+ | 7.60 | |
| AA | 7.30 | 18 |
| AA- | 7.00 | 15 |
| A+ | 6.85 | 24 |
| A | 6.65 | 42 |
| A- | 6.40 | 38 |
| BBB+ | 6.25 | 38 |
| BBB | 5.85 | 59 |
| BBB- | 5.65 | 52 |
| BB+ | 5.25 | 34 |
| BB | 4.95 | 25 |
| BB- | 4.75 | 65 |
| B+ | 4.50 | 78 |
| B | 4.15 | 115 |
| B- | 3.75 | 95 |
| CCC+ | 3.20 | 23 |
| CCC | 2.50 | 10 |
| CCC- | 1.75 | 6 |
| D | 0.00 | 14 |

Average based on over 750 U.S. industrial corporate with rated debt outstanding; 1994 data.

Source: In-Depth Data Corporation

bond rating equivalents calculated at each quarterly interval.¹¹

All that is necessary is that the issuing firm (or borrower) was operating for the entire sample period, e.g., five years, and had quarterly financial statements. The actual bonds/loans did not have to be outstanding in the period, as is necessary when returns and variance of returns are used. Since the actual debt issue may not have been outstanding during the entire measurement period, leverage measures will likely also vary over time. Still, we expect to capture most of the covariance of default risk between firms, although not the actual overlap (joint probability region) of default (see footnote 11 above).

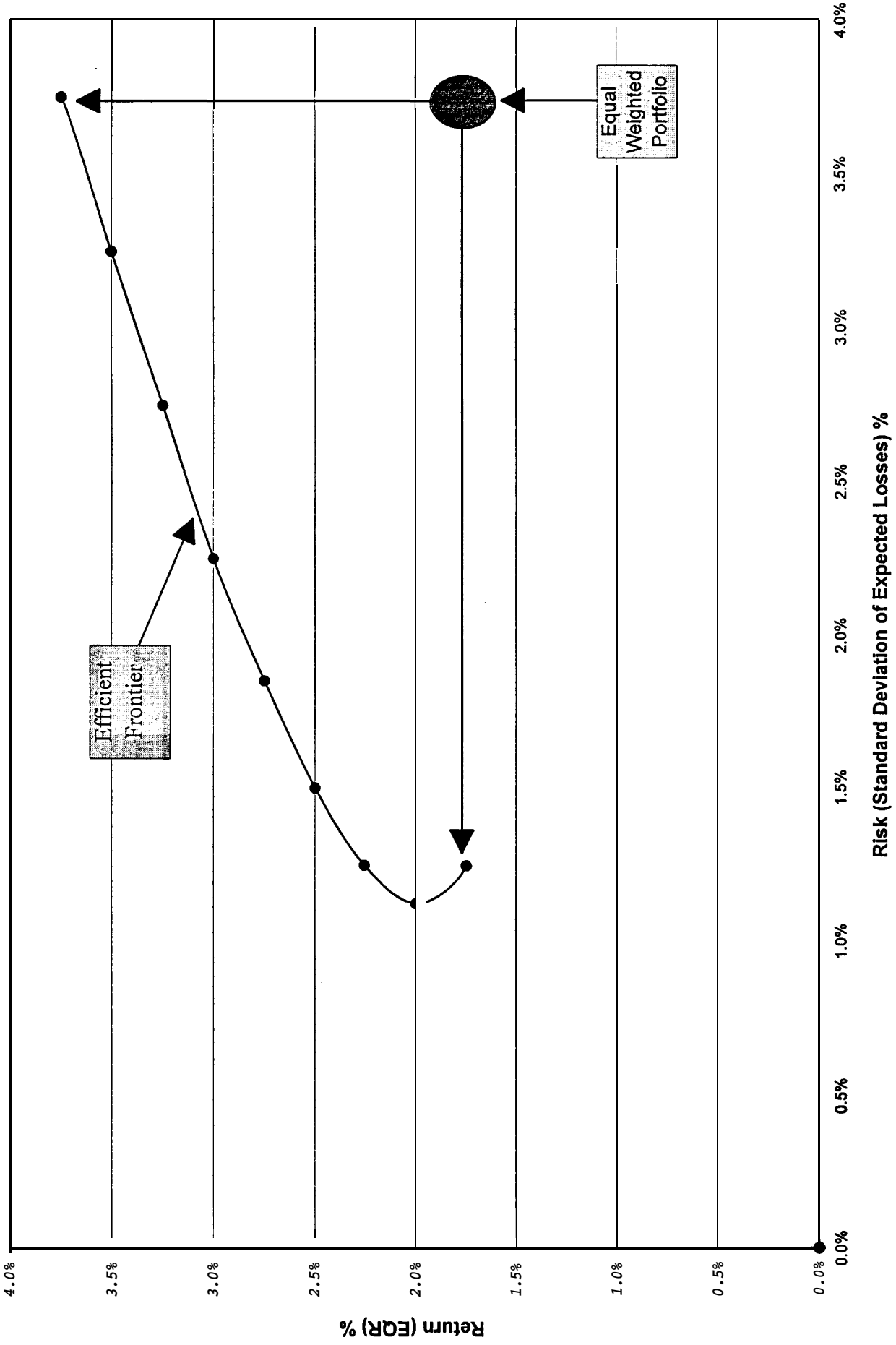
Empirical Results

We ran the portfolio optimizer program¹² on the same ten bond portfolio analyzed earlier, this time using the Z"-Score (EM Score) bond rating equivalents and their associated expected and unexpected losses instead of returns. Exhibit 7 shows the

¹¹We do recognize that our measure of covariance is potentially biased in two ways. First, estimates of individual firms' debt unexpected losses are derived from empirical data on bonds from a given bond rating class and as such will probably understate the risk of loss from individual firm defaults. On the other hand, the covariance of default losses between two firms' debt could be analyzed as being based on the joint probability of both defaulting at the same time. If the default decision of each firm is viewed as 0,1, i.e., as a binomial distribution, then the appropriate covariance or correlation should be calculated from a joint density function of two underlying binomial distributions. Our measure, however, assumes a normal density function for returns and thus returns are jointly, normally distributed for each firm which could result in a higher aggregate measure of portfolio risk. As such, the two biases neutralize each other to some extent although it is difficult to assess the relative magnitude of each.

¹²Using a double precision, linear constrained, optimization program (DLCONG).

Exhibit 7
Efficient High Yield Bond Portfolio, Using EMS Scores
10 Issues



efficient frontier compared to an equal weighted portfolio. As we observed earlier, the efficient frontier indicates considerably improved HYPRs. For example, the return/risk ratio of just above 0.50 (1.75/3.4) for the equal weighted 10-bond portfolio can be improved to 1.60 (2.00/1.25) at the 2.00% quarterly return level and to about 1.00 for the same risk (3.75%) level.

Exhibit 8 shows the portfolio weights for the efficient frontier portfolio using both returns and risk (unexpected losses) when the individual weights are constrained at a maximum of 15% of the portfolio.¹³ This is for the 1.75% quarterly expected return. Note that both portfolios utilize eight bonds out of ten and very similar weightings. Indeed, seven of the eight bonds appear in both portfolios. These results are comforting in that the unexpected loss derived from the Z"-Score is an alternative risk measure. Our small sample test results are encouraging and indicate that this type of portfolio approach is potentially quite feasible for fixed income assets. The important factor in our analysis is that credit risk management plays a critical role in the process.

We should note clearly that these are preliminary findings. Subsequent conceptual refinements and larger sample empirical tests are necessary to gain experience and confidence with this portfolio technique for fixed income assets (including loans).

¹³The unconstrained weighting results yielded efficient portfolios of between five and eight individual bonds with some weightings of over 30%. These high weights would not be prudent for most portfolio managers.

Exhibit 8

Return=1.75%

Constrained To 15% Maximum Weights

| Company Ticker | Weights Using Zeta Scores | Weights Using Returns (Quarterly) |
|-------------------|------------------------------|--------------------------------------|
| AS | <i>0.0000</i> | 0.1065 |
| BOR | 0.0776 | <i>0.0000</i> |
| CGP | 0.1500 | 0.1500 |
| CQB | 0.1500 | 0.1500 |
| FA | 0.0000 | 0.0000 |
| IMD | 0.1500 | 0.1351 |
| RHR | 0.1500 | 0.1209 |
| STO | 0.1500 | 0.1500 |
| USG | 0.1500 | 0.1500 |
| WS | 0.0224 | 0.0376 |

Source: Data for this analysis was generously supplied by the Global Corporate Bond Research Department of Salomon Brothers Inc.

Conclusion

In this paper we have presented a new approach to measure the return-risk trade-off in portfolios of risky debt instruments, whether bonds or loans. The use of complex, statistically based portfolio techniques to manage assets of financial institutions and fixed income portfolio money managers is very much in its early phase and will continue to evolve, perhaps more quickly in the near future. Our approach substitutes the concept of unexpected loss for the more traditional variance of return measure used in equity securities analysis. Preliminary empirical tests indicates some reason to be optimistic about this approach.

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in corporate bond credit analysis. 30.1 Credit ratings. A credit rating is a formal opinion given by a rating agency, of the credit risk for investors in a particular issue of debt. 501. Chapter 30: Corporate Bonds and Credit Analysis. Other ratios that are considered include: intangibles, that is the portion of intangibles relative to the asset side of a balance sheet. Cohen, J., Zinbarg, E., Zeikel, A., Investment Analysis and Portfolio Management, Richard D Irwin, 1977. S&P, Credit Week, 8 November 1993, Standard & Poor's. Strategic Value Analysis, Mars Business Associates. Thus "commercial bank investment policy emerges from a straight forward application of the theory of portfolio management to the particular circumstances of commercial bank." Portfolio management refers to the prudent management of a bank's assets and liabilities in order to seek some optimum combination of income or profit, liquidity, and safety. When a bank operates, it acquires and disposes of income-earning assets. These assets plus the bank's cash make up what is known as its portfolio. It is not a theory but simply a method to analyse a borrower's creditworthiness. It gives the bank criteria for evaluating the potential of a borrower to successfully repay a loan on- time. 1 Financial Institutions Center Corporate Bond and Commercial Loan Portfolio Analysis by Edward I. Altman 96-41. 2 THE WHARTON FINANCIAL INSTITUTIONS CENTER The Wharton Financial Institutions Center provides a multi-disciplinary research approach to the problems and opportunities facing the financial services industry in its search for competitive excellence. Anthony M. Santomero Director The Working Paper Series is made possible by a generous grant from the Alfred P. Sloan Foundation. 3 Corporate Bond and Commercial Loan Portfolio Analysis 1 September 1996 Abstract: In this paper we have presented a new approach to measure the return-risk trade-off in portfolios of risky debt instruments, whether bonds or loans. Corporate bonds are debts issued by industrial, financial, and service companies to finance capital investments. There is wide range of choices for corporate bonds in terms of bond structures, coupon rates, maturity dates, credit quality, and industry exposure. A short-term corporate bond has a maturity of less than 5 years, an intermediate bond has a maturity of 5-10 years, and long-term bond have a maturity of 10 years. Corporate bonds are rated by one or more primary rating agencies, including Standard and Poor's, Moody's, and Fitch. Bond ratings are based on various financial parameters of